SELECTION OF A SUITABLE ORTHOGONAL WAVELET AND ITS OPTIMUM LEVEL OF DECOMPOSITION FOR NOISE REDUCTION IN DIGITAL WIRELESS COMMUNICATION

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ABSTRACT: Wireless communication has become one of the fastest growing fields in today's technological world. Growth in technology has led to unprecedented demand for high speed architectures for complex signal processing applications. In digital wireless communication system the reliability of Orthogonal Frequency Division Multiplexing (OFDM) is limited because of the time-varying nature of the channel. To avoid the effect of ICI(Inter Carrier Interference) cyclic prefix guard intervals were introduced before each block of parallel data symbols, but these guard intervals decreases the spectral efficiency of the OFDM system. To avoid the insertion of guard intervals wavelet based OFDM (OWDM) are used which gives a fine spectral containment of the signal over a wide range of the Bandwidth.

Keywords: OFDM, Wavelet, Decomposition, Reconstruction, Optimum level.

I. INTRODUCTION

Wireless communication has become one of the fastest growing fields in today's technological world. Starting from the very beginning, this was always a matter of curiosity to know how of this field is becoming more and more important? Unlike wired- line communications, wireless communication is also burdened with many challenges such as, scarcity of the Radio Spectrum, propagation-path loss due to diversity of propagation media and bit-error rate. However the greatest achievement of wireless communication systems is the support for personal mobility, and it has emerged as a platform to support information to users on the move. Support for mobility has changed the dimensions of wireless communication and has evolved as the base of mobile communication.

For wireless communication systems, limited bandwidth allocations coupled with a potentially large pool of users restrict the bandwidth availability for other users. As time passes the demand for high performance, high capacity and high bit rate wireless communication systems, to integrate wide variety of communication services such as high-speed data, video and multimedia traffic as well as voice signals. These demands have been fulfilled by use of Orthogonal Frequency Division Multiplexing (OFDM), which provides an efficient means to handle high-speed data streams over a multipath fading environment. Conventionally, the implementation of OFDM uses Fourier filters for data modulation and demodulation, via the inverse fast Fourier Transform (IFFT) and fast Fourier transform (FFT) respectively. But these Fourier filters have larger size and poor flexibility. Deep study suggests that, implementation of wavelet based OFDM (OWDM) provides performance gains over Fourier based OFDM due to better spectral containment properties of wavelet filters. All the characteristics of OFDM modulated signals are directly dependent on the set of waveforms of which it makes use. Several authors have foreseen "wavelet theory" as a good platform, on which to build OFDM waveform bases.

Wavelet transform is a tool for studying signals in the joint time-frequency domains. Which is capable of providing the time and frequency information simultaneously, thus giving a time-frequency representation of the signal. Wavelets are known to have compact support (localization) both in time and frequency domain, and possess better orthogonality. The Wavelet transform is a way of decomposing a signal of interest into a set of basis waveforms, called wavelets, which thus provide a way to analyse the signal by examining the coefficients (or weights) of wavelets. This method is used in various applications and is becoming very popular among technologists, engineers and mathematicians alike. In most of the applications, the power of the transform comes from the fact that the basis functions of the transform are

localized in time (or space) and frequency, and have different resolutions in these domains. Different resolutions often correspond to the natural behaviour of the process one wants to analyse, hence the power of the transform. these properties make wavelets and wavelet transform natural choices in fields as diverse as image synthesis, data compression, computer graphics and animation, human vision, radar, optics, astronomy, acoustics, seismology, nuclear engineering, biomedical engineering, magnetic resonance imaging, music, fractals, turbulence and pure mathematics. Recently wavelet transform has also been proposed as a possible analysis system when designing sophisticated digital wireless communication systems, with advantages such as transform flexibility, lower sensitivity to channel distortion and interference and better utilization of spectrum. Wavelets have found beneficial applicability in various aspects of wireless communication systems design including channel modelling, transceiver design, data representation, data compression, and source and channel coding, interference mitigation, signal de-noising and energy efficient networking.

II. WAVELET TRANSFORM

The transform of a signal is just another form of representing the signal. In Fourier theory a signal can be represented as the sum of a possibly infinite series of sinusoids, which is referred to as a Fourier expansion. Fourier expansion works well with time invariant signals. For a time-varying signal, a complete characterization in the frequency domain should include the time aspect, resulting in the time-frequency analysis of a signal. Wavelet analysis is a form of "multi-resolution analysis", which means that wavelet coefficients for a certain function contain both frequency and time-domain information. This fact makes wavelets useful for signal processing applications where knowledge of both frequency information and the location in time of that frequency information is useful.

The wavelet transform is a multi-resolution analysis mechanism where an input signal is decomposed into different frequency components, and then each component is studied with resolutions matched to its scales. The Fourier transform also decomposes signals into elementary waveforms, but these basis functions are sines and cosines. Thus, when one wants to analyse the local properties of the input signal, such as edges or transients, the Fourier transform is not an efficient analysis tool. By contrast the wavelet transforms which use irregularly shaped wavelets offer better tools to represent sharp changes and local features. The wavelet transform gives good time resolution and poor frequency resolution at high frequencies and a good frequency resolution and poor time resolution at low frequencies. This approach is logical when the signal on hand has high frequency components for short durations and low frequency components for long durations. Fortunately,

the signals that are encountered in most engineering applications are often of this type.
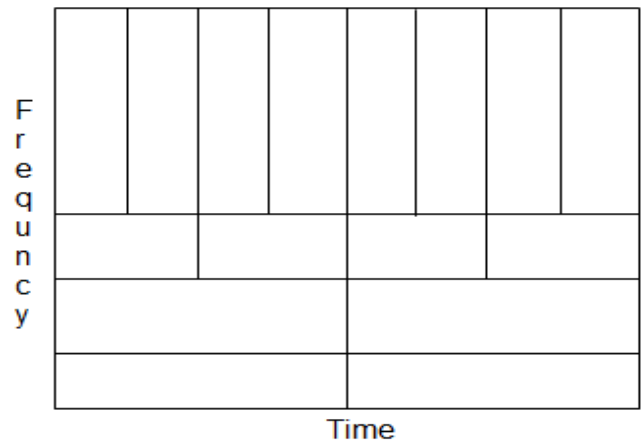

Fig.1 Time-frequency tiling of the wavelet transform

Every block in the figure corresponds to one value of the wavelet transform in the time-frequency plane. All the points in the time-frequency plane that fall into a block are represented by one coefficient of the wavelet transform. It is easy to infer from the figure that at lower frequencies the width and height of the windows are long and short, giving good frequency resolution and poor time resolution. At higher frequencies the width and height of the windows are short and long respectively giving good time resolution and poor frequency resolution

A. MOTIVATION FOR USING WAVELETS FOR WIRELESS COMMUNICATION

❖ Wavelet transform can create subcarriers of different bandwidth and symbol length.

❖ The ability of wavelets to arrange the time-frequency tiling in a manner that minimizes the channel disturbances minimizes the effect of noise and interference on the signal.

❖ Wavelets give a new dimension, signal diversity which could be exploited in a cellular communication system, where adjacent cells can be designated different wavelets in order to minimize inter-cell interference.

❖ Wavelet-based algorithms have long been used for data compression. By compressing the data, a reduced volume of data is transmitted so that the communication power needed for transmission is reduced.

❖ Design of the pulse shape. Many researchers have proven that the wavelet based multi-carrier schemes are superior in suppressing ICI and ISI as compared to the traditional Fourier based systems.
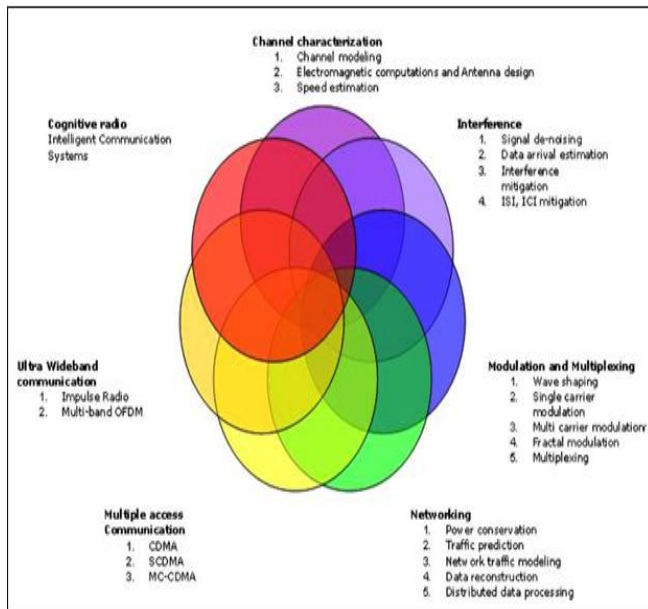

Fig.2 The Spectrum of wavelet application for wireless communication

III. CONTINUOUS AND DISCRETE WAVELET TRANSFORM

Wavelet Transforms are broadly classified as Continuous and discrete wavelet transform.

A. CONTINUOUS WAVELET TRANSFORM (CWT)

The Continuous Wavelet Transform (CWT) is used to represent a continuous time signal in terms of shifted and scaled versions of a prototype wavelet function $\psi(t)$. The function $\psi(t)$, known as the basic (or mother) wavelet function, plays a vital role in CWT similar to that played by sinusoidal wave in Fourier Transform. The CWT of a continuous time signal $x(t)$ is the set of coefficients $X_{CWT}(s,\tau)$ for $s \in (-\infty,\infty), \tau \in (-\infty,\infty)$. The coefficients are obtained using the inner product operation between $x(t)$ and $\psi(t)$ as follows:

$$X_{CWT}(s,\tau) = \int x(t)\psi_{s,\tau}(t)\,dt \tag{1}$$

The wavelet *basis function* $\psi_{s\tau}(t)$ is defined in terms of the mother wavelet function as:

$$\psi_{s\tau}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right) \tag{2}$$

and is sometimes called *baby wavelet* [5]. In the above equation "s", is the transformed *compressing* or *stretching* it and "τ", the *translation* parameter along the time-axis. The scale parameter "s", is defined as 1/ *frequency* and corresponds to frequency information of the signal. The translation parameter "τ", relates to the location of the wavelet as it is shifted through the signal and it corresponds to the time information of the signal in the wavelet transform. The factor $\frac{1}{\sqrt{|s|}}$ is for energy normalization across the different scales. Wavelet modulation has a novel multi-rate diversity strategy that offers improved message recovery over conventional modulation techniques: if the message is not received at one rate due to channel disturbances, it can be received at another rate where the channel is clear.

The original signal can be reconstructed using the Inverse wavelet Transform:

$$X_{ICWT}(t) = \frac{1}{c_\psi}\iint \frac{1}{s^2}X_{CWT}(s,\tau)\psi_{s,\tau}(t)\,ds\,d\tau \tag{3}$$

B. Discrete Wavelet Transform (DWT)

Wavelets are the wave forms with desirable characteristics of localization both in time and frequency. They also possess the property of orthogonality across scale and translation. The discrete wavelet transform (DWT) provides a means of decomposing sequences of real numbers in a basis of compactly supported orthonormal sequences each of which is related by being a scaled and shifted version of a single function. As such it provides the possibility of efficiently representing those features of a class of sequences localized in both position and scale and possess the property of orthogonality across scale and translation.

The DWT analyses the signal at different frequency bands with different resolutions by decomposing the signal into an approximation containing coarse and detailed information. DWT employs two sets of functions, known as scaling and wavelet functions, which are associated with low pass and high pass filters. The decomposition of the signal into different frequency bands is simply obtained by successive high pass and low pass filtering of the time domain signal. The original signal $x[n]$ is first passed through a half-band high pass filter $g[n]$ and a half-band low pass filter $h[n]$. A half-band low pass filter removes all frequencies that are above half of the highest frequency, while a half-band high pass filter removes all frequencies that are below half of the highest frequency of the signal. The low pass filtering halves the resolution, but leaves the scale unchanged. The signal is then sub-sampled by two since half of the number of samples is redundant, according to the Nyquist's rule.
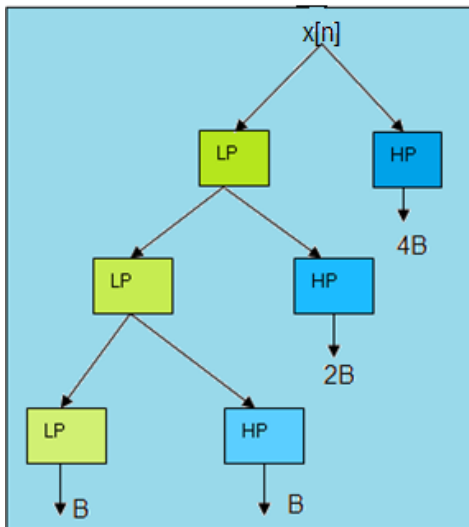
Fig.3. Decomposition of input signals[10]

This decomposition can be mathematically be expressed as follows:

$$y_{high}[k] = \sum_n x[n]g[2k - n] \tag{4}$$

$$y_{low}[k] = \sum_n x[n]h[2k - n] \tag{5}$$

Where $y_{high}[k]$ and $y_{low}[k]$ are the outputs of the high pass and low pass filters, after up-sampling and down- sampling by a factor of two.

This decomposition halves the time resolution since only half the number of samples then comes to characterize the entire signal. Conversely it doubles the frequency resolution, since the frequency band of the signal spans only half the previous frequency band effectively reducing the uncertainty by half. The above procedure, which is also known as sub-band coding, can be repeated for further decomposition. At every level, the filtering and sub-sampling will result in half the number of samples (and hence half the time resolution) and half the frequency bands being spanned (and hence doubles the frequency resolution). This procedure is illustrated in Figure 3, where $x[n]$ is the original signal to be decomposed, and $h[n]$ and $g[n]$ are the respective impulse responses of the low pass and high pass filters.

IV. Sub-band Coding

Sub-band coding is a hierarchical coding scheme where the signal to be coded is successively split into high and low frequency components. The wavelet transform can be regarded as a form of sub-band coding where the signal to be analysed is passed through a series of filter banks. The outputs of the different filter stages are then the wavelet and scaling function transform coefficients.

The filter bank is built by using filters that iteratively split the spectrum into two equal parts, high pass and low pass. The high-pass parts contain the smallest details and hence are not to be processed any further. However, the low-pass

part still contains some details and therefore it is split again. This dyadic operation is repeated until the required degree of resolution is obtained. Usually the number of sub-bands is limited by the amount of data or computation power available. The process of splitting the spectrum is graphically displayed in Figure 4.

The scaling functions and wavelet functions are associated with low pass and high pass filters, respectively. Figure 3 shows the decomposition of a discrete time signal $x(n)$ into different frequency bands i.e. scaling and wavelet functions by successive high pass and low pass filtering $x(n)$. The original signal $x(n)$ is first passed through a high pass filter $g(n)$ and a low pass filter $h(n)$ and decimated by 2.The output of different filter stages are the wavelet and scaling functions transform coefficient .
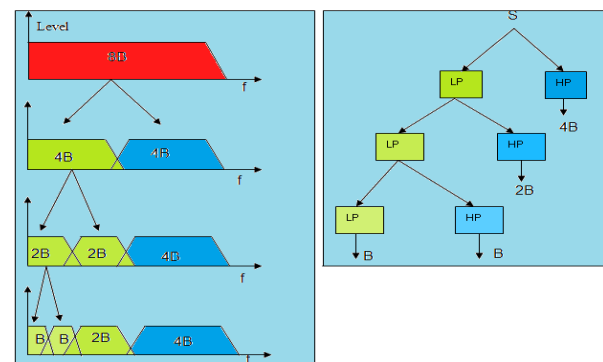
Fig. 4: Splitting the signal spectrum with an iterated filter bank [10]

V. SELECTION OF A SUITABLE WAVELET

The selection of a suitable wavelet for digital wireless communication depends on its length and shape of the signal. The chosen wavelet must be of shortest duration and close to the analysed signal. The chosen Wavelet must be Orthogonal with multi-rate filters. The orthogonality is important because [9]:

(i)   It implies that energy content of the signal is preserved through the wavelet transform; therefore transient signals are not missing any information through the transform.

(ii)   It allows for multi-resolution analysis (MRA), therefore, the WT have different analysis in different scales to extract high and low frequency details of the transient signals

(iii)   The inner product of the signal with the orthonormal basis obtains the wavelet coefficient.

The necessary and sufficient conditions for orthonormality are

$$\sum h(m)h(m - 2n) = \delta(n) = \begin{cases} 1 & if \ n = 0 \\ 0 & otherwise \end{cases} \tag{6}$$

$$\sum g(m)h(m - 2n) = 0 \tag{7}$$

where m, n $\in$ Z, and Z is a positive integer.

To ensure the orthogonality and perfect reconstruction h(n) and g(n) must satisfy the following condition:

$$g[n] = \pm(-1)^n h[N - n] \qquad (8)$$

where n is odd and N $\in$ Z.

VI. WHY ORTHOGONAL BEHAVIOUR OF SIGNAL IS NECESSARY FOR DIGITAL WIRELESS COMMUNICATION

A signal is a vector and a vector can be represented as a sum of its components in a variety of ways, depending upon the choice of coordinate system.

A. *Components of a vector:*

A vector is having both; the magnitude and the direction .
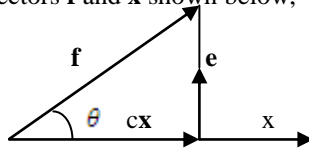For two vectors **f** and **x** shown below,

fig. (a)

f = c**x** + **e**
$$\qquad (9)$$

their dot (inner or scalar) product as

$$f.x = |f||x| \cos\theta \qquad (10)$$

Where θ is angle between these vectors .
Let the component of **f** along **x** be **cx**. Geometrically the component of **f** along **x** is the projection of **f** on **x** and is obtained by drawing a perpendicular from the tip of **f** on the vector **x.**
Equation (9) is not the only way to express **f** in terms of **x**. The figure shows the infinite other possibilities.

fib (b)   fig(c)
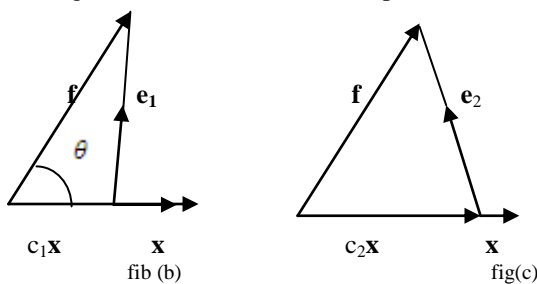
$$\mathbf{f} = c_1\mathbf{x} + \mathbf{e}_1 = c_2\mathbf{x} + \mathbf{e}_2 \qquad (11)$$
In each of these three representations **f** is represented in terms of **x** plus another vector called **error vector**.
From eq. (9)
e = **f** – c**x**   is the error in the approximation in the signal.
$$\qquad (12)$$
From here we find that , to reduce the error between two signals the projection of f along x should be minimum, which is zero in this case. And this is possible only when f and x are perpendicular (Orthogonal) to each other.
Also we can have

$$c = \frac{f.x}{x.x} = \frac{1}{|x|^2} f.x \qquad (13)$$

Hence for f and x to be orthogonal the inner (scalar or dot) product of the two vectors is zero, that is if
    f . x = 0   (14)

VII. FINDING THE OPTIMUM LEVEL OF WAVELET DECOMPOSITION
The maximum level to apply the wavelet transform depends on how many data points contain in a data set, since there is a down-sampling by 2 operations from one level to the next one. In our experience, one factor that affects the number of level can reach to achieve the satisfactory noise removal results is the signal-to-noise ratio (SNR) in the original signal. To calculate this value of SNR, Matlab simulation is used. The objective of this research is to find the optimum level of wavelet decomposition to get the better performance for reducing noise in wireless communication.

VII. Methodology
The methodology we adopt to get optimum level of wavelet decomposition is to calculate the value of SNR during the process of modulation and demodulation technique in wireless communication.
We used Daubechies (Db) and Coiflet (coif) wavelet decomposition with level of 2,4,6, and 8 The noisy signals are de-noised at different levels of wavelet decomposition. SNR is measured at for each level of wavelet decomposition. In this study, optimum level is obtained by comparing SNR for successive simulation with different level of Daubechies, and Coiflet wavelet decompositions.

SNR is basically a dimensionless quantity, which is a ratio of signal power to the noise power contained in the signal. The noise, $N(t)$ observed in the transmitted signal, $x(t)$ has a stochastic description in communication technology. The power of a deterministic signal is given by

$$P_s = \int_0^T x^2(t)dt$$
$$\qquad (15)$$
Where T is the time period of the signal.
If the signal is a stationary stochastic process , its power is defined as the value of its correlation function $R_s(\tau)$ at the origin
$$R_s(\tau) = E[s(t)s(t+\tau)]; \qquad P_s = R_s(0)$$
$$\qquad (16)$$
Here $E[.]$ denotes the expected value. The noise power, P_N is related to its correlation function.
$$P_N = R_N(0)$$
$$\qquad (17)$$
Thus SNR is determined according to the power of transmitted signal, $x(t)$ and noise, $N(t)$ in communication technology.
$$SNR = \frac{P_s}{P_N}$$
$$\qquad (18)$$

In this objective, SNR is estimated through the parameter of mean value and standard deviation taken from the residual options in wavelet tools. SNR is defined as the ratio of mean to standard deviation of a signal.

$$SNR = \frac{\mu}{\sigma}$$

(20)

Where μ the signal is mean or expected value and σ is the standard deviation of the noise.

The parameters of histogram and cumulative histogram are considered in the study to get the precise decomposition level.

IX. Results and Discussion

The signal to noise ratio (SNR) for different wavelet decomposition is measured for un-scaled white noise. All simulation results of different wavelet decompositions are provided as a function of signal to noise ratio (SNR).

The result of simulation for different level of Daubechies and Coiflet wavelet decomposition are shown in table 1.

Table 1.

Wavelet	Level Decomposition	Mean(μ)	Standard Deviation(σ)	SNR=$\frac{\mu}{\sigma}$ in dB
Db2	2	0.7795	1.837	-3
	4	0.1701	2.028	-10
	6	0.03681	2.04	-17
	8	0.006207	2.04	-25
Coif2	2	0.7843	1.768	-3
	4	0.1743	1.963	-10
	6	0.03792	1.975	-17
	8	0.006935	1.975	-24

From this table it is clearly observed that both of these two wavelets didn't give any satisfactory result regarding level of wavelet decomposition. When the mean value is considered we find considerable decrement.

As a result SNR performance is not very good .As we go on increasing the level of wavelet decomposition.

X. Conclusion:

Optimum level of wavelet decomposition shows better performance in noisy and multipath fading channels. The Optimum wavelet decomposition level is obtained by measuring the SNR for reducing noises in wireless transmission system.
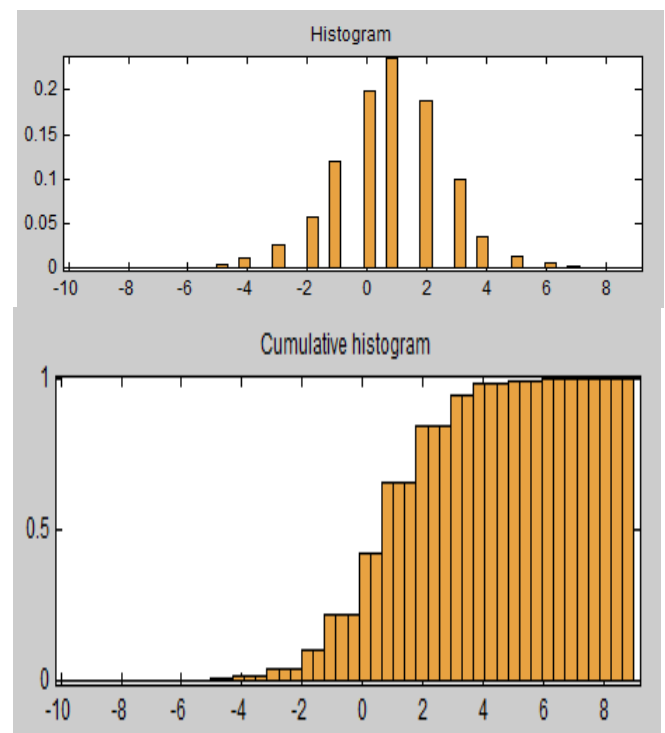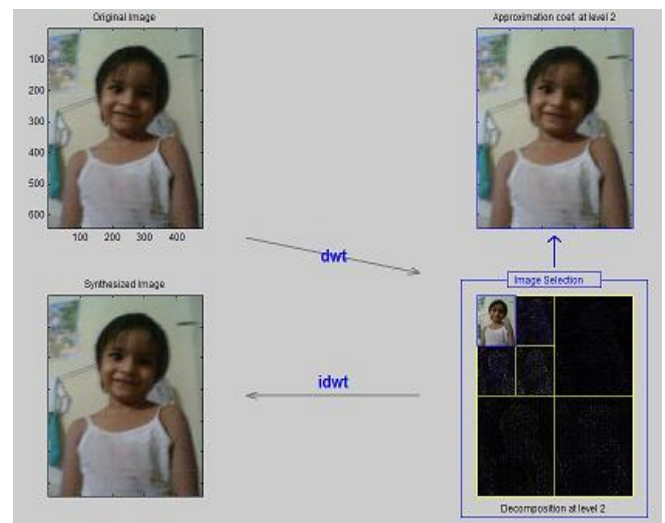

Fig.5 Histogram and Cumulative histogram for 2 level decomposition

References

[1] M. K. Lakshmanan. Nikookar, "A Review of Wavelets for Digital Wireless Communication"Wireless Personal Communications Vol. 37, Issue: 3-4, pages 387–420, Springer 2006.

[2] R.S. Bodhe, S.S. Narkhede, Shirish Joshi, "Design and Implementation of Baseband Processing for Wavelet OFDM", National Conference e-PGCON 2012, Pune.

[3] Lakshmanan M. K., & Nikookar, H. (2007,September).Wavelet packet based strategy to mitigate wideband interference on impulse radio. In *18th international symposium on personal, indoor and mobile radio communications (PIMRC)*, Athens, Greece.

[4] Burrus, C. S., Gopinath, R. A., & Guo, H. (1998). *Introduction to wavelets and wavelet transforms, a primer*. Upper Saddle River, NJ: Prentice Hall.

[5] Daubechies, I. (1992). *Ten lectures on wavelets*. Philadelphia: SIAM

[6] Lindsey, A. (1997, May).Wavelet packet modulation for orthogonally trans-multiplexed communications. *IEEE Transaction On Signal Processing, 45*, 1336–1339.

[7] S.L. Linfoot, "Wavelet families for orthogonal wavelet division multiplex", Electronics Letters, Vol.44, No.18, pp.1101-1102, August 2008.

[8] J. G. Proakis, "Digital Communication", 3rd ed., New York, Mc-Graw-Hill, 1995

[9] Uwe Ladebusch and Claudia A. Liss, "Terrestrial DVB (DVB-T): A Broadcast Technology for Stationary Portable and Mobile Use", IEEE Proceeding, Vol.94, pp. 183-193, 1st January, 2006.

[10] S.L. Linfoot, "Wavelet families for orthogonal wavelet division multiplex", Electronics Letters, Vol.44, No.18, pp.1101-1102, August 2008

Biography

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